SIMULATING THE DISTRIBUTION OF AXON SIZE IN NERVES

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ABSTRACT

The electrical excitation threshold of a nerve axon is dependent upon the physical properties of the axon membrane as well as the axon diameter. Histological studies have confirmed that peripheral afferent muscle nerves have a characteristic distribution of axon diameters. We approximate a probability density function for this distribution that consists of the summation of several Gaussian functions. A transformation technique is then applied for the random generation of the nerve axon diameters from uniformly distributed random numbers. For the example presented, the resultant distribution of peripheral afferent muscle nerve axon diameters conforms approximately to the characteristic distribution that has been observed experimentally.

INTRODUCTION

In simulating the electrical excitation of nerves, it is desirable to have a realistic model of the excitation thresholds of the individual axons. Since the excitation threshold to electrical stimulation for an axon is dependent upon the size of the axon, a technique for randomly generating axon diameters that conform to experimentally observed distributions would be useful.

We apply a technique for generating random numbers that conform to an arbitrary probability density function from a uniformly distributed random number generator [1]. This transformation technique is useful in a wide variety of applications since many random number generators available in commonly used numerical software packages generate numbers that are uniformly distributed between a given range.

As an example, we focus on the characteristic distribution of axon diameters found in peripheral afferent muscle nerves. We approximate a probability density function for this distribution that consists of the summation of four Gaussian functions [2]. The probability density function of the axon diameter random variable is illustrated in Figure 1.

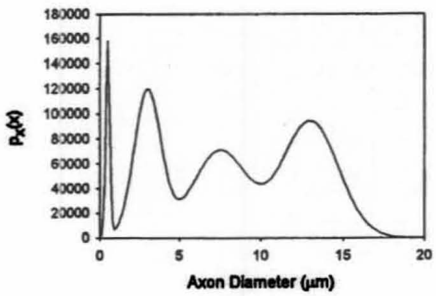

Figure 1. Peripheral afferent muscle nerve axon diameter probability density function. The curve is composed of the summation of four Gaussian functions with means, amplitudes and standard deviations as shown in Table 1.

An expression (1) can be written for the probability density function shown in Figure 1. where x is a random variable representing the axon diameter. The expression consists of four Gaussian functions with parameters defined in Table 1.

$$p_X(x) = \frac{\Psi_1}{\sigma_1\sqrt{2\pi}} e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}} + \frac{\Psi_2}{\sigma_2\sqrt{2\pi}} e^{-\frac{(x-\mu_2)^2}{2\sigma_2^2}} + \frac{\Psi_3}{\sigma_3\sqrt{2\pi}} e^{-\frac{(x-\mu_3)^2}{2\sigma_3^2}} + \frac{\Psi_4}{\sigma_4\sqrt{2\pi}} e^{-\frac{(x-\mu_4)^2}{2\sigma_4^2}} \quad (1)$$

	MODE 1	MODE 2	MODE 3	MODE 4
Ψ	0.05	0.25	0.30	0.40
μ (μm)	0.5	3.0	7.5	13.0
σ (μm)	0.1274	0.8493	1.699	1.699

Table 1. Gaussian function parameters as per equation (1). These parameters were used to generate the probability density function illustrated in Figure 1.

TRANSFORMATION CALCULATION PROCEDURE

We outline a procedure that is an application of a technique commonly used in the simulation of communications systems [1]. The initial step in the procedure is to obtain the cumulative distribution function from the probability density function of (1). This step is most conveniently achieved through the use of numerical integration techniques. The numerically calculated cumulative distribution function is illustrated in Figure 2.

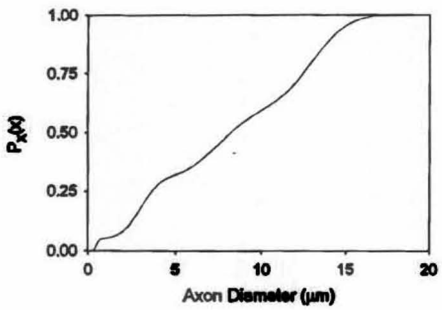

Figure 2. Cumulative distribution function of peripheral nerve axon diameters calculated from the probability density function of Figure 1.

Given the cumulative distribution function of a random variable x which is written as $P_X(x)$ and a random variable y that is uniformly distributed, x can be obtained from y as per the transformation shown in (2).

$$x = P_X^{-1}(y) \tag{2}$$

This relationship is conveniently applied when a closed form expression is available for the inverse of the cumulative distribution of x. The transformation can still be used when no closed form expression exists. In this case, interpolation techniques are used to find the value of the random variable x from the cumulative distribution function that corresponds to a uniformly distributed random value of y between zero and one.

SIMULATION RESULTS

MATHCAD [3] was used to calculate the cumulative distribution function from (1). The uniformly distributed random number generator was then used and its values were interpolated on the cumulative distribution function shown in Figure 2.

Figure 3 represents the normalized histogram of 5000 random numbers that have been generated using the probability density function of Figure 1. as per the parameters of Table 1.

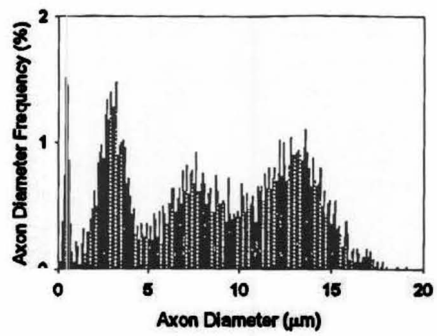

Figure 3. Normalized histogram of 5000 uniformly distributed random numbers generated between zero and one that have been transformed according to the probability density function of (1).

DISCUSSION

We have presented an application of a random variable transformation that allows for the generation of random numbers that conform to a specific probability density function using a uniformly distributed random number generator. In this case, the technique has been used to generate random numbers that represent axon diameters with a distribution consistent with histological studies of the axon sizes found in peripheral afferent muscle nerves. The technique can be applied to generating random values for axon diameters that are consistent with the distribution found in efferent motor nerves or nerves with other distributions.

This research was supported by a grant from the Natural Sciences and Engineering Research Council of Canada (NSERC).

REFERENCES

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